Analytical Study on the Modulation Instability of the Relativistic Plasma

دراسة تحليلية لعدم الاستقرار المعدل في البلازما النسبية

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2003 ١٤٢٤

جامعة الأندلس

للعلوم والتقنية

Alandalus University For Science & Technology

(AUST)

ISSN : 2410-7727

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Abstract :

In the current study the researchers attempt to investigate the growth of the modulation instability and soliton propagation in the high-temperature non-collisional homogeneous plasma. Analytical relations of nonlinear Schrodinger equation coefficients are proposed for phase velocity ranges which are greater than or equal to light velocity.

The current findings confirm those of the study conducted by Wahdain and Daraigan [2014] when the phase velocity ranges , 1) exceed the velocity of light and not even close to, and 2) are equal to the velocity of light [11].

This study has found out that the modulation instability occurs at all values of phase velocity that are greater than or equal to light velocity, and the growth rate of the modulation instability reaches it is maximum value when the phase velocity ranges are sufficiently close to the velocity of light.

Keywords: Nonlinear Schrodinger equation, Relativistic plasma , Instability modulation , Growth rate.

الملخص العربي :

في البحث الحالي تم دراسة نمو عدم الاستقرار المعدل وانتشار السوليتون في البلازما المتجانسة اللاتصادمية عالية الحرارة، وتم استنباط علاقات تحليلية لمعاملات معادلة شرودنجر اللاخطية تغطي كافة مديات سرع الطور الضوئية و فوق الضوئية.

نتائج الدراسة الحالية تؤكد النتائج التي توصل إليها الباحثان [Wahdain and Daraigan, 2014] عند مديات سرعة الطور، 1) أكبر من سرعة الضوء وليست قريبة منها وأيضاً، 2)عند سرعة الطور المساوية لسرعة الضوء .

أوضحت الدراسة الحالية أن عدم الاستقرار المعدل يحدث عند جميع سرع الطور أكبر من أو تساوي سرعة الضوء، ويأخذ معدل نمو عدم الاستقرار المعدل قيمة عظمى عند سرع الطور القريبة من سرعة الضوء .

الكلمات الدالة: معادلة شرودنجر اللاخطية، البلازما النسبية، عدم الاستقرار المعدل، معدل النمو.

ISSN : 2410-7727

Introduction :

The modulation instability of Langmuir waves is based on nonlinear Schrodinger equation estimated from both Vlasov relativistic equation and Maxwell equations [8 , 12] by which valid results are produced at different temperature levels (high and low) . It has been studied by many researchers (such as Wahdain and Daraigan,2014 ; Kotov et al. 1984 ; Pataraya and Melikidze, 1980 ; Timofeev, 2013 ; Krafft and Volokitin, 2010 ; Hakimi Pajouh et al.,2004, among others) .

These studies are limited at the calculation of the effect of nonlinear interaction to the grow of modulation instability , and the soliton propagation due to the difficulty of finding the coefficients integration mentioned in nonlinear Schrodinger equation to compute approximate solutions. The grow of modulation instability is also studied by [3 , 4 , 8] at high wave length . Furthermore, nonlinear Schrodinger equation for light waves has been studied by [5 , 8 , 9 , 11] at phase velocity equal to the velocity of light. Others studied it at the phase velocity which is greater than velocity of light and not even close to it , [11] .

However, there is no analytical formula for nonlinear Schrodinger equation coefficients which includes the phase velocity between $(\omega / k = c)$ and $(\omega / k > c)$. Hence, the main objective of this research is to estimate analytical relation for nonlinear Schrodinger equation coefficients. This would help to follow the growth on the modulation instability and soliton propagation in the non-collisional homogeneous plasma at high temperature for the range phase velocity $(\omega / k \geq c)$, whose linear Landau damping is small value exponentially or not existing .

Theoretical Aspect:

To solve the problem, the relativistic Vlasov equation and Maxwell equations are used. The equations are written down in the dynamical frame of reference where the coordinate x′ and t′ are

ISSN : 2410-7727

connected with the coordinate x and the time t of the laboratory frame of reference using Lorentz's relation as follows:

$$x' = \epsilon\left(x - \upsilon_g\, t\right)\ ; \qquad t' = \epsilon\left(t - \frac{\upsilon_g}{c^2}x\right) \qquad (1)$$

where $\epsilon = \left(1 - \frac{v_g^2}{c^2}\right)^{-\frac{1}{2}}$, and υ_g is the group velocity of the high frequency linear waves in the laboratory frame of reference .

All the values of the equations are presented by the formula

$$F=F^{(0)} + \sum_{\substack{n=1\\ l=1}}^{\infty} \zeta^n F_n^l\,(\xi\,,\tau)\,\exp\,[il(\,k'x' - \omega't')] \quad (2)$$

Where $F^{(0)}$ is the unperturbed meaning of the function F. k' and ω' are the wave vector and the frequency of the linear waves in the dynamical frame of reference , $i = \sqrt{-1}$, $\zeta \ll 1$ is the small parameter and

$$\xi = \zeta x', \qquad\qquad \tau = \zeta t' \qquad (3)$$

In the laboratory frame of reference the unperturbed distribution function of the particles is taken as the relativistic Maxwell function .

The mathematical transformation used in [2; 8; 12] are adopted to obtain the equation for the first harmonic of intensity of electric field in the form:

$$i\frac{\partial E}{\partial \tau} + P\frac{\partial^2 E}{\partial \xi^2} + q|E|^2E + \frac{s}{\pi}pv\int_{-\infty}^{+\infty}\frac{|E(\xi')|^2}{\xi - \xi'}d\xi' E = 0 \ldots\ldots (4)$$

where (pv) denotes the Cauchy principle value , the term which included the parameter (s) depending on the nonlinear Landau damping .

The nonlinear Schrodinger equation coefficients take the fallowing formulas :

$$P = \frac{1}{2}\left[v_g^2\frac{\partial^2\varepsilon}{\partial\omega^2} + 2\upsilon_g\frac{\partial^2\varepsilon}{\partial\omega\,\partial k} + \frac{\partial^2\varepsilon}{\partial k^2}\right]\left(\frac{\partial\varepsilon}{\partial\omega}\right)^{-1} \qquad \ldots (5)$$

$$q = \mathrm{Re}\ Q \qquad ; \qquad s = \mathrm{Im}\,Q \ \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

ISSN : 2410-7727

$$Q = \left[L(\omega, k, \delta z) - \frac{M^2(\omega, k)}{6} - \frac{N^2(\omega, k, \delta z)}{C(\delta z)} \right] \left(\frac{\partial \varepsilon}{\partial \omega} \right)^{-1} \text{.........} \quad (7)$$

$$\upsilon_g = -\left(\frac{\partial \varepsilon}{\partial k}\right)\left(\frac{\partial \varepsilon}{\partial \omega}\right)^{-1} = k^{-1}\left(\omega - 2\left(\frac{\partial \varepsilon}{\partial \omega}\right)^{-1}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots . \quad (8)$$

$$L(\omega, k, \delta z) = -\sum_{\sigma} 2\pi e_{\sigma}^4 c \int d^3 p \left(\frac{3 p_z p_{0\perp}}{p_0^5 \Delta^4} - \frac{k c p_{0\perp}^4}{p_0^6 \Delta^5} - \frac{2 p_{0\perp} \delta k c}{p_0^6 \Delta^4 \delta\Delta} \right) \frac{\partial f_{0\sigma}(\vec{p})}{\partial p_z} \dots (9)$$

$$M(\omega, k) = \sum_{\sigma} 4\pi e_{\sigma}^3 c \int d^3 p \frac{p_{0\perp}}{p_0^3 \Delta^3} \frac{\partial f_{0\sigma}(\vec{p})}{\partial p_z} \dots\dots\dots\dots\dots\dots\dots (10)$$

$$N(\omega, k) = \sum_{\sigma} 4\pi e_{\sigma}^3 c \int d^3 p \frac{p_{0\perp} \delta k c}{p_0^3 \Delta^4 \delta\Delta} \frac{\partial f_{0\sigma}(\vec{p})}{\partial p_z} \dots\dots\dots\dots\dots\dots\dots (11)$$

$$C(\delta z) = \sum_{\sigma} 4\pi e_{\sigma}^2 \int d^3 p \frac{\delta k c}{\delta\Delta} \frac{\partial f_{0\sigma}(\vec{p})}{\partial p_z} \dots\dots\dots\dots\dots\dots (12)$$

where σ : is the particle type ; k and ω are the wave vector and the frequency of the Langmuir waves . $\varepsilon = \varepsilon(\omega, \kappa)$ is the longitudinal dielectric constant of non-collisional homogenous plasma ; $f_{0\sigma}(\vec{p})$ is the unperturbed meaning of relativistic Maxwell distribution function ; $p_0 = \sqrt{p^2 c^2 + m_0^2 c^4}$ the total energy of particle a p its momentum ; m_0 : the rest mass.

$$\delta z = \frac{\delta\omega}{\delta kc}; \ \delta\Delta = \delta\omega - \delta k v_z; \ \Delta = \omega - k v_z; \ p_{0\perp}^2 = p_0^2 - p_z^2 .$$

A solution of the nonlinear Schrodinger equation (1) is unstable if the Lighthill's condition ($Pq > 0$), so the small deviations condition lead for strengthening the pulses and wave packet pressure.

The maximum value of the growth rate of the modulation instability for the perturbation in the identical direction with the direction of nonlinear wave propagation take the following formula:

$$\gamma_{max} = (q^2 + s^2)^{1/2} E_0$$

and the wave number for the perturbation is equal:

ISSN : 2410-7727

$$k_{max} = [(q^2 + s^2)/Pq]^{1/2} E_0$$

If the Lighthill's condition was satisfied and $|q| \gg |s|$, strengthening of the pulses occurs and an increase in pressure on the packet wave, so the nonlinear Schrodinger equation (4) can be solved in the form of Soliton [8, 11] :

$$E = (2E_0)^{1/2} \operatorname{sech}\left[E_0 (q/P)^{1/2} (x - wt)\right] \exp\left\{ i \left[\frac{w}{2P}\left(x - \frac{wt}{2}\right) - qE_0^2 t \right]\right\} \ldots\ldots\ldots.. (13)$$

Where: w is the optional constant. In this case $w = \upsilon_g$.

Taking the integration of the function $\varepsilon(\omega, k)$; $L(\omega, k, \delta z)$; $M(\omega, k)$; $N(\omega, k, \delta z)$; $C(\delta z)$ by angles in spherical coordinates of space momentum , so that the oz-axis is identical with the direction wave vector $\vec{k}$. Then, taking the integration by parts, and in light of the distribution function which describes the plasma equilibrium (the relativistic Maxwell distribution function) (see[11]), we are able to express these values by the following functions [10]:

$$G^{(0)}(\alpha, x) = 2x \int_1^\infty \frac{\exp(-\alpha u_0)}{(x^2 - 1)u_0 + 1} \cdot \frac{du_0}{u} \;\; ; G^{(n)}(\alpha, x) = \frac{\partial^n}{\partial \alpha^n} G^{(0)}(\alpha, x) \ldots .(14)$$

where:

$$(x = z, \delta z) \;\; ; z = \omega / kc \; ; u_0 = \frac{p_0}{mc^2} = \sqrt{1 + u^2} \; ; u = \frac{p}{mc} ; \alpha = mc^2 / T$$

The above-mentioned functions (14) are used in the linear theory of plasma [10]. Yet, these functions will be applied to the non-linear theory of plasma to find out analytical relations for Schrodinger equation coefficients which include values of phase velocity that are greater than or equal to light velocity.

In order to complete the calculations, we should find the integration in (5) so that we get the waves of light $(x = z = 1)$:

$$G^{(0)}(\alpha, z = 1) = 2K_0(\alpha)$$

where: $K_0(\alpha)$ MacDonald zero order function [1] .

While taking the integration of non-light waves $(x \neq 1)$ requires special treatment as it is difficult to calculate it in a general form, for that we express the function $G^{(0)}(\alpha, x)$ in the following form:

$$G^{(0)}(\alpha, x) = -x\mu\left[F^{+}\exp(\alpha\mu) - F^{-}\exp(-\alpha\mu)\right] \qquad (15)$$

where:

$$F^{\pm} = \int_{1}^{\infty} \exp(-\alpha(u_0 \pm \mu))(u_0 \pm \mu)^{-1} \frac{du_0}{u} \quad ; \quad \mu = (1 - x^2)^{-1/2}$$

After taking derivative of a function $F^{\pm}$ by a variable α, an integration by the variable u_0 then $G^{(0)}(\alpha, z)$ and $G^{(0)}(\alpha, \delta z)$ taking the form:

$$\begin{pmatrix} G^{(0)}(\alpha, z) \\ G^{(0)}(\alpha, \delta z) \end{pmatrix} = \begin{pmatrix} \ln\left(\frac{z+1}{z-1}\right) & \cos(\alpha\beta) \\ \ln\left(\frac{1+\delta z}{1-\delta z}\right) & ch(\alpha\delta\beta) \end{pmatrix} - \pi\begin{pmatrix} \sin(\alpha\beta) \\ i\, sh(\alpha\delta\beta) \end{pmatrix} + 2\begin{pmatrix} z\beta \\ -\delta z\delta\beta \end{pmatrix} \int_0^{\alpha} K_0(\xi) \begin{pmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \end{pmatrix} d\xi \qquad (16)$$

where:

$$\beta = (z^2 - 1)^{-1/2} \quad ; \quad \delta\beta = (1 - \delta z^2)^{-1/2} \; ; \; \mathrm{I}_1 = \sin[\beta(\alpha - \xi)] \; ; \mathrm{I}_2 = sh\,[\delta\beta\,(\alpha - \xi)]$$

By using function expansion $K_0(\xi)$ [1]

$$K_0(\xi) = \left(\ln\left(\frac{2}{\xi}\right) - \gamma\right)\sum_{n=0}^{\infty}\frac{(\xi^2/4)^n}{(n!)^2} + \sum_{m=1}^{\infty}\frac{(\xi^2/4)^m}{(m!)^2}\left(\sum_{k=1}^{m} 1/k\right) \qquad (17)$$

(Euler constant: $\gamma = 0.577216$) can be expression $G^{(0)}(\alpha, x)$ as:

$$G^{(n)}(\alpha, x) = G_0^{(n)}(\alpha, x) + \Delta^{(n)}\, G(\alpha, x)$$

where:

$$\begin{pmatrix} G^{(0)}(\alpha, z) \\ G^{(0)}(\alpha, \delta z) \end{pmatrix} = \begin{pmatrix} \ln\left(\frac{z+1}{z-1}\right) \cos(\alpha\beta) \\ \ln\left(\frac{1+\delta z}{1-\delta z}\right) ch(\alpha\delta\beta) \end{pmatrix} - \pi\begin{pmatrix} \sin(\alpha\beta) \\ i\, sh(\alpha\delta\beta) \end{pmatrix} + 2\begin{pmatrix} z\beta \\ -\delta z\delta\beta \end{pmatrix} \int_0^{\alpha} \left(\ln\left(\frac{2}{\xi}\right) - \gamma\right) \begin{pmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \end{pmatrix} d\xi \qquad (18-a)$$

$$\begin{pmatrix} G_0^{(1)}(\alpha, z) \\ G_0^{(1)}(\alpha, \delta z) \end{pmatrix} = \frac{\partial}{\partial\alpha}\begin{pmatrix} G_0^{(0)}(\alpha, z) \\ G_0^{(0)}(\alpha, \delta z) \end{pmatrix} ; \begin{pmatrix} G_0^{(2)}(\alpha, z) \\ G_0^{(2)}(\alpha, \delta z) \end{pmatrix} = \frac{\partial^2}{\partial\alpha^2}\begin{pmatrix} G_0^{(0)}(\alpha, z) \\ G_0^{(0)}(\alpha, \delta z) \end{pmatrix} + \begin{pmatrix} z\beta \\ -\delta z\delta\beta \end{pmatrix} \int_0^{\alpha} \left(\ln\left(\frac{1}{\xi}\right)\right) \begin{pmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \end{pmatrix} d\xi \qquad (18-b)$$

ISSN : 2410-7727

$$G_0^{(3)}(\alpha,z)=\frac{\partial}{\partial\alpha}G_0^{(2)}(\alpha,z)\ ;\ G_0^{(4)}(\alpha,z)+\frac{\partial^2}{\partial\alpha^2}G_0^{(2)}+\frac{3}{4}z\beta\int_0^{\alpha}\ln\left(\frac{1}{\xi}\right)\mathrm{I}_1\,d\xi\ ..(18-c)$$

$$\begin{pmatrix}\Delta^{(n)}G^{(0)}(\alpha,z)\\ \Delta^{(m)}G^{(0)}(\alpha,\delta z)\end{pmatrix}=2\begin{pmatrix}z\beta\\ -\delta z\delta\beta\end{pmatrix}\int_0^{\alpha}\Delta^{\binom{n}{m}}K(\alpha-\xi)\begin{pmatrix}\sin\beta\xi\\ sh\,\delta\beta\xi\end{pmatrix}d\xi\ ;\begin{pmatrix}n=0,1,2,3,4.\\ m=0,1,2.\end{pmatrix}.(19)$$

$$\Delta^{(0)}K(\xi)=K_0(\xi)-\ln\left(\frac{2}{\xi}\right)\ ;\ \Delta^{(1)}K(\xi)=\frac{\partial\Delta^{(0)}K(\xi)}{\partial\xi}\ ;$$

$$\Delta^{(2)}K(\xi)=\frac{\partial^2\Delta^{(0)}K(\xi)}{\partial\xi^2}-\frac{1}{2}\ln\left(\frac{1}{\xi}\right)=M+\delta_1K(\xi)\ ;\ \Delta^{(3)}K(\xi)=\frac{\partial\Delta^{(2)}K(\xi)}{\partial\xi}=\delta_2K(\xi)$$

$$\Delta^{(4)}K(\xi)=\frac{\partial^2\Delta^{(2)}K(\xi)}{\partial\xi^2}-\frac{3}{8}\ln\left(\frac{1}{\xi}\right)=N+\delta_3K(\xi)\ ;$$

$$M=const.\,;N=const.\,;\ \Delta^{(0)}K(0)=\Delta^{(1)}K(0)=\delta_1K(0)=\delta_2K(0)=\delta_3K(0)=0$$

The exactly form of the function $\Delta^{(n)}K(\xi)$ can be found in equation (17).

After that we will take the integration in equations (18), the values of $G_0^{(m)}(\alpha,\delta z),G_0^{(n)}(\alpha,z)$ take the following forms:

$$G_0^{(0)}(\alpha,z)=2z\left(\ln\left(\frac{2}{\alpha}\right)-\gamma-g(\alpha\beta)\right)+\pi(z-1)\sin(\alpha\beta)+a\cos(\alpha\beta).(20)$$

$$G_0^{(1)}(\alpha,z)=\beta\{-2z\,f(\alpha\beta)+\pi(z-1)\cos(\alpha\beta)-a\sin(\alpha\beta)\}........(21)$$

$$G_0^{(2)}(\alpha,z)=\beta^2\{2z\,g(\alpha\beta)+\frac{\pi}{2}(z+2)(z-1)^2\sin(\alpha\beta)-(a+b)\cos(\alpha\beta)\}+z\left(\ln\left(\frac{1}{\alpha}\right)-g(\alpha\beta)\right)(22)$$

$$G_0^{(3)}(\alpha,z)=\beta^3\left\{2z\left(f(\alpha\beta)-\frac{1}{\alpha\beta}\right)+\frac{\pi}{2}(z+2)(z-1)^2\cos(\alpha\beta)+(a+b)\sin(\alpha\beta)\right\}-z\beta\,f(\alpha\beta)..(23)$$

$$G_0^{(4)}(\alpha,z)=\beta^4\{z\left(2(\alpha\beta)^{-2}+(z-3)g(\alpha\beta)\right)+\frac{\pi}{8}(z-1)^3(3z^2+9z+8)\sin(\alpha\beta)+$$
$$\left[a+b\left(1-\frac{3}{4}\left(z^2-1\right)\right)\right]\cos(\alpha\beta)\}+\frac{3}{4}\left(\ln\left(\frac{1}{\alpha}\right)-g(\alpha\beta)\right)..............................(24)$$

ISSN : 2410-7727

$$G_0^{(0)}(\alpha,\delta z) = -i\pi\exp(-\alpha\delta\beta) + \ln\left(\frac{1+\delta z}{1-\delta z}\right)ch(\alpha\delta\beta) + \delta z\left[2\left(\ln\left(\frac{2}{\alpha}\right)-\gamma\right) - 2\ln(2\delta\beta)ch(\alpha\delta\beta) + E_i(-\alpha\delta\beta)\exp(\alpha\delta\beta) + E_i^*(\alpha\delta\beta)\exp(-\alpha\delta\beta)\right] \qquad (25)$$

$$G_0^{(1)}(\alpha,\delta z) = i\pi\delta\beta\exp(-\alpha\delta\beta) + \delta\beta\ln\left(\frac{1+\delta z}{1-\delta z}\right)sh(\alpha\delta\beta) + \delta z\delta\beta\left[E_i(-\alpha\delta\beta)\exp(\alpha\delta\beta) - E_i^*(\alpha\delta\beta)\exp(-\alpha\delta\beta) - 2\ln(2\delta\beta)sh(\alpha\delta\beta)\right] \qquad (26)$$

$$G_0^{(2)}(\alpha,\delta z) = -i\pi\delta\beta^2\exp(-\alpha\delta\beta) + \delta\beta^2\ln\left(\frac{1+\delta z}{1-\delta z}\right)ch(\alpha\delta\beta) + \delta z\delta\beta^2\left[E_i(-\alpha\delta\beta)\exp(\alpha\delta\beta) + E_i^*(\alpha\delta\beta)\exp(-\alpha\delta\beta) - 2\ln(2\delta\beta)ch(\alpha\delta\beta)\right] \qquad (27)$$

where:

$$a = (z-1)\ln(z-1) + (z+1)\ln(z+1) - 2z\ln(2)\,;\; b = z\beta^{-2}(\gamma - 0.5\ln(z+1) - 0.5\ln(z-1))$$

$$E_i(-x) = \int_{-\infty}^{-x} t^{-1}e^t dt \;;\; E_i^*(x) = \int_{-\infty}^{x} e^t t^{-1} dt \;;\; g(x) = -Ci(x)\cos(x) - si(x)\sin(x);$$

$$f(x) = Ci(x)\sin(x) - si(x)\cos(x) \;;\; si = Si(x) - \pi/2;$$

$$Si(x) = \int_0^x t^{-1}\sin(t)\,dt \;;\; Ci(x) = \gamma + \ln x + \int_0^x (\cos(t)-1)t^{-1}\,dt.$$

At high temperature ($\alpha = \dfrac{mc^2}{T} \langle\langle 1$) , the functions become as: $\Delta^{(n)}G(\alpha,z)$ and $\Delta^{(n)}G(\alpha,\delta z)$ can be neglected because it has very small and limited values. The following duo evaluations of these functions have been obtained by taking the integration by parts of the equation (19):

$$\left|\Delta^{(n)}G(\alpha,z)\right| \le \min\left\{4z\left|\Delta^{(n)}K(\alpha)\right|;\; 2z(\alpha\beta)^2\left|\Delta^{(n)}K(\alpha)\right|\right\}; n = 0,1,2,3,4 \qquad (28-a)$$

$$\left|\Delta^{(0)}G(\alpha,\delta z)\right| \le 2\delta z\left|\Delta^{(0)}K(\alpha)\right|(ch(\alpha\delta\beta)-1); \qquad (28-b)$$

$$\left|\Delta^{(1)}G(\alpha,\delta z)\right| \le 2\delta z\delta\beta\left|\Delta^{(1)}K(\alpha)\right|sh(\alpha\delta\beta)\,; \qquad (28-c)$$

$$\left|\Delta^{(2)}G(\alpha,\delta z)\right| \le 2\delta z\delta\beta^2\left|\Delta^{(2)}K(\alpha)\right|ch(\alpha\beta) \qquad (28-d)$$

It should be taken into account that the value in the downomnater of the relativistic Maxwell distribution function [11] at high temperatures will be as follows:

ISSN : 2410-7727

$$\left(2\alpha K_2(\alpha)\right)^{-1} \approx (1+\nu)\alpha/4\,;\ \nu \approx \alpha^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

Based on equations (19) and (29) and duo evaluation of the functions $\Delta^{(n)}G(\alpha,z)$ and $\Delta^{(n)}G(\alpha,\delta z)$ the relations of the values $\varepsilon(\omega,k), L(\omega,k,\delta z), M(\omega,k), N(\omega,k,\delta z), C(\delta z)$ of relativistic plasma at high temperatures $(\alpha\langle\langle 1)$ can be written as follows:

$$\varepsilon(\omega,k) = 1 + Z_p^2\,\alpha - \frac{1}{4} Z_p^2\, z\alpha\left(2G_0^{(0)}(\alpha,z) - 2\alpha\, G_0^{(1)}(\alpha,z) + \alpha^2 G_0^{(2)}(\alpha,z)\right)\ldots(30)$$

$$L(\omega,k,\delta z) = \frac{1}{8}\left(\frac{Z_p}{kc}\right)^2\left(\frac{e}{mc}\right)^2 \alpha^4\Big[dG_0^{(1)}(\alpha,z) - hG_0^{(3)}(\alpha,z) - l\frac{\partial}{\partial z} G_0^{(1)}(\alpha,z) +$$

$$p\frac{\partial}{\partial z} G_0^{(3)}(\alpha,z) + r\frac{\partial^2}{\partial z^2} G_0^{(1)}(\alpha,z) - t\frac{\partial^2}{\partial z^2} G_0^{(3)}(\alpha,z) - w\frac{\partial^3}{\partial z^3} G_0^{(1)}(\alpha,z) +$$

$$y\frac{\partial^3}{\partial z^3} G_0^{(3)}(\alpha,z) - \frac{1}{\alpha}\,{}^{\&}e^{\&}\, G_0^{(0)}(\alpha,z) + \eta G_0^{(0)}(\alpha,\delta z)\Big]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(31)$$

$$M(\omega,k) = -\frac{1}{4} Z_p^2 \frac{e}{mc}\frac{\alpha^2}{kc}\Big[\alpha^{-2}\frac{5z^2-1}{4z}\left(G_0^{(4)}(\alpha,z) - G_0^{(2)}(\alpha,z)\right) + \frac{(z^2-1)^2}{4z}\frac{\partial}{\partial z}$$

$$\left(G_0^{(4)}(\alpha,z) - G_0^{(2)}(\alpha,z)\right) - 1.5\, z\alpha^{-2}\left(G_0^{(0)}(\alpha,z) - G_0^{(1)}(\alpha,z)\right)\Big]\ldots\ldots\ldots(32)$$

$$N(\omega,k,\delta z) = -Z_p^2 \frac{e}{mc}\frac{\alpha^2}{2}\Big[1 + 0.5\frac{z^2-1}{z-\partial z}\alpha^2\left(G_0^{(4)}(\alpha,z) - G_0^{(2)}(\alpha,z)\right) - 0.5\frac{2z^3 - 3z^2\delta z + \delta z}{(z-\delta z)^2}$$

$$\left(G_0^{(0)}(\alpha,z) - \alpha\, G_0^{(1)}(\alpha,z)\right) + 0.5\frac{\delta z(1-\delta z^2)}{(z-\delta z)^2}\left(G_0^{(0)}(\alpha,\delta z) - \alpha\, G_0^{(1)}(\alpha,\delta z)\right)\Big]\ldots\ldots\ldots(33)$$

$$C(\delta z) = \omega_p^2\left(\alpha - \left(\frac{\alpha\delta z}{4}\right)\right)(2G_0^{(0)}(\alpha,\delta z) - 2\alpha G_0^{(1)}(\alpha,\delta z) + \alpha^2 G_0^{(2)}(\alpha,\delta z))\ldots(34)$$

where: $z_p = \dfrac{\omega_p}{kc}$; $\omega_p = \sqrt{\dfrac{4\pi e^2 n}{m}}$: plasma frequency , e: is the electron charge and n: is the plasma density m: is mass of particle and the parameters $\left(d,h,l,p,r,t,w,y,{}^{\&}e^{\&},\eta\right)$ are algebraic functions in z and δz. The functions $G_{(0)}^{(n)}(\alpha,z)$ and $G_{(0)}^{(n)}(\alpha,\delta z)$ are defined in equations (20)- (27).

 ISSN : 2410-7727

In the two limiting cases, the forms (30)-(34) become as:

The phase velocities are greater than velocity of light and not even very close to it :

All the functions included in the defined values $\varepsilon(\omega,k)$, $L(\omega,k,\delta z)$, $M(\omega,k)$, $N(\omega,k,\delta z)$ and $C(\delta z)$ are related with the parameters $(\alpha\beta)$ or $(\alpha\delta\beta)$. In this limiting case , $\alpha\beta \langle\langle 1$ and $\alpha\delta\beta \langle\langle 1$., and by using the following relations [1]:

$$Si(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!} \quad ; \quad Ci(x)=\gamma+\ln x+\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{2n(2n)!} \;;$$

$$\{E_i(-x)\ ;\ E_i^*(x)\}=\gamma+\ln x+\sum_{n=1}^{\infty}\{1\,;\ (-1)^n\}\frac{x^n}{n.n!}$$

the following form has been calculated according to the evaluation of the equation (28), and (30) – (34):

$$\varepsilon(\omega,k)=1+Z_p^2\,\alpha\left(\frac{z}{2}\ln\left(\frac{z+1}{z-1}\right)-1\right)$$

$$\operatorname{Re}L(\omega,k,\delta z)=\frac{1}{8}Z_p^2\left(\frac{e}{kmc^2}\right)^2\alpha^3\left\{\frac{4z\beta^{-2}}{(z-\delta z)^3}-\frac{30z^4-46z^2+8}{3(z^2-1)^2}-4\frac{4z^2-1}{(z-\delta z)^2}+\right.$$
$$8z\frac{9z^2-7}{3(z-\delta z)}\beta^2-\frac{2\delta z\delta\beta^{-4}}{(z-\delta z)^4}\ln\left(\frac{1+\delta z}{1-\delta z}\right)-$$
$$\left.\frac{z^5-2z^4\delta z-2z^3\delta z^2+8z^2\delta z^3-7z\delta z^4+4\delta z^3-2\delta z}{(z-\delta z)^4}\ln\left(\frac{z+1}{z-1}\right)\right\}$$

$$\operatorname{Im}L(\omega,k,\delta z)=\frac{\pi}{4}\frac{\omega_p^2}{(kc)^4}\left(\frac{e}{mc}\right)^2\alpha^3\frac{\delta z\delta\beta^{-4}}{(z-\delta z)^4}\exp(-\alpha\delta\beta)$$

$$M(\omega,k)=\frac{1}{2}\left(\frac{\omega_p}{kc}\right)^2\frac{e}{kmc^2}\alpha^2\left[\frac{3}{2}z\ln\left(\frac{z+1}{z-1}\right)-\frac{3z^2-2}{z^2-1}\right]$$

$$\operatorname{Re}N(\omega,k,\delta z)=\frac{1}{2}\left(\frac{\omega_p}{kc}\right)^2\frac{e}{mc}\alpha^2\left[\frac{\delta z-2z}{z-\delta z}-\frac{1}{2}\frac{\delta z\delta\beta^{-2}}{(z-\delta z)^2}\ln\left(\frac{1+\delta z}{1-\delta z}\right)+\right.$$
$$\left.\frac{1}{2}\frac{2z^3+\delta z-3z^2\delta z}{(z-\delta z)^2}\ln\left(\frac{z+1}{z-1}\right)\right]$$

ISSN : 2410-7727

$$\operatorname{Im} N(\omega,k,\delta z)=\frac{\pi}{2}\frac{\omega_p^2}{(kc)^2}\frac{e}{mc}\alpha^2\frac{\delta z\delta\beta^{-2}}{z-\delta z}\exp(-\alpha\delta\beta)$$

$$\operatorname{Re} C(\delta z)=\omega_p^2\alpha\left(1-\frac{1}{2}\delta z\ln\left(\frac{1+\delta z}{1-\delta z}\right)\right)+F\ ;\quad \operatorname{Im} C(\delta z)=0.5\pi\,\omega_p^2\,\alpha\,\delta z\exp(-\alpha\,\delta\,\beta)$$

Where :

$$F=\begin{cases}\omega_{p_i}/\delta z^2 \ldots\ldots\ldots\ldots\ldots\ldots\ \delta k\upsilon_{T_i}\langle\langle\,|\delta\omega|\langle\langle\,\delta k c\\ c^2 r_{D_i}^{-2}\ \ldots\ldots\ldots\ldots\ldots\ldots\ |\delta\omega|\langle\langle\,\delta k\,\upsilon_{T_i}\end{cases}$$

$\omega_{pi}=\sqrt{\frac{4\pi e^2 n_{0i}}{m_i}}$: ion plasma frequency ,$r_{Di}$: Debye radius ; v_{Ti}: ion sound velocity .

It is the same values that have been obtained by Wahdain and Daraigan, 2014 [11]

The phase velocities are equal to the velocity of light ($z=\frac{\omega}{kc}=1$) :

In this case first we can find the approach solutions of the values $\varepsilon(\omega,k)$, $L(\omega,k,\delta z)$, $M(\omega,k)$, $N(\omega,k,\delta z)$ and $C(\delta z)$ of the variable $(\alpha\beta\rangle\rangle 1)$ according to the equations (30) -(34) , and then we can

put $z=1$ and substitute in the relations (5) - (8) which obtain the following coefficients::

$$\omega^2=\omega_p^2\alpha\ln(1.85/\alpha)\quad ;\quad \upsilon_g=c\left[1-\frac{\alpha^2}{6}\ln(1.85/\alpha)\right];$$

$$P=\frac{5}{6}\delta\beta^{-3}\frac{c^2\alpha^2}{\omega}\ln(0.89/\alpha)[\ln(1.85/\alpha)]^{\frac{3}{2}}$$

$$\begin{pmatrix}q\\-s\end{pmatrix}=\left(\frac{e}{mc}\right)^2\omega^{-2}\begin{pmatrix}q_1+q_2+q_3\\s_1+s_2\end{pmatrix}$$

ISSN : 2410-7727

The above-calculated coefficients correspond to the coefficients nonlinear Schrodinger equation obtained by Wahdain and Daraigan, 2014 [11].

Discussion and Conclusions :

In this study, we have found the analytical formula of the coefficients of nonlinear Schrodinger equation covers all ranges of phase velocity which are greater than or equal to light velocity $(\omega/k \geq c)$. With the help of these, we can trace the growth of the modulation instability and soliton propagation in the non-collisional homogeneous plasma at high temperature.

The derived relations (30) - (34) are suitable for numerical calculations because they contain known simple and special functions.

It is clear that the relations which have been obtained from (30) - (34) in both limiting cases $(\alpha\beta \langle\langle 1)$ and $(z = 1)$ tend to match with the obtained results in [11].

We have traced the numerical values q and s for the ratios of the variable $(\alpha\beta)$ which are shown in Figures (1) and (2) , where we have inferred that the modulation instability occurs at all ranges of phase velocity which are greater than or equal to light velocity $(\omega/k \geq c)$.

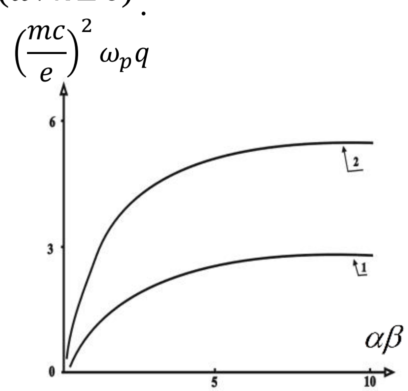

Fig. (1): Relationship between q and $\alpha\beta$, electron-ion plasma.

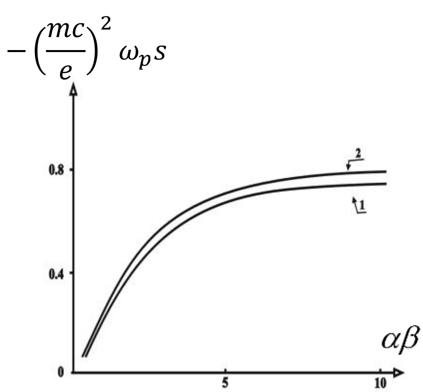

Fig. (2): Relationship between S and $\alpha\beta$, electron-ion plasma,.

ISSN : 2410-7727

We also arrived to the conclusion that the modulation instability takes a maximum value of the growth rate at phase velocity when it is close to the velocity of light. Meanwhile, the modulation instability growth rate of the long-wave is decreased when the temperature increases, and vice-versa in regard with short waves.

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